INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2010; **63**:548–563
Published online 3 June 2009 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.2087

Analytical treatment on magnetohydrodynamic (MHD) flow and heat transfer due to a stretching hollow cylinder

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SUMMARY

This paper studied on magnetohydrodynamics flow and heat transfer outside a stretching cylinder. Momentum and energy equations are reduced using similarity transformation and converted into a system of ordinary differential equations which are solved analytically by the homotopy analysis method. The effects of the parameters involved, namely the magnetic parameter (M), Prandtl number (Pr) and Reynolds number (Re) on the velocity and temperature fields are investigated.

The obtained results are valid for the whole solutions' domain with high accuracy. These methods can be easily extended to other linear and nonlinear equations and so can be found widely applicable in engineering and sciences. Copyright © 2009 John Wiley & Sons, Ltd.

Received 4 February 2009; Revised 7 April 2009; Accepted 9 April 2009

KEY WORDS: homotopy analysis method (HAM); MHD flow; heat transfer; boundary layer; stretching cylinder; analytical treatment

1. INTRODUCTION

Some industrial equipments such as magnetohydrodynamic (MHD) generator, pumps, bearings and boundary layer control are affected by the interaction between the electrically conducting fluid and a magnetic field. The works of many investigators have been studied in relation to these applications.

One of the basic and important problems in this area is the hydromagnetic behavior of boundary layers along fixed or moving surfaces in the presence of a transverse magnetic field. MHD boundary layers are observed in various technical systems employing liquid metal and plasma flow transverse of magnetic fields [1].

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Recently, many researchers have studied the influences of electrically conducting fluids, such as liquid metals, water mixed with a little acid and others in the presence of a magnetic field on the flow and heat transfer of a viscous and incompressible fluid past a moving surface or a stretching plate in a quiescent fluid. Pavlov [2] was one of the first pioneers in this study. After Pavlov the flow past a moving flat plate or a stretching sheet in the presence of a transverse magnetic field convert an interesting subject that an amount of literature has been generated on this problem [3–13]. Examples of such technological applications are hot rolling, wire drawing, glass–fibre and paper production, drawing of plastic films, metal and polymer extrusion, metal spinning, liquid films in condensation process, etc. [13]. In all these cases, it is important to investigate cooling and heat transfer for the improvement of the final products. Because many properties of final products depend to a large extent on the skin friction coefficient and on the surface heat transfer rate. However, to the best of our knowledge, only Wang [14] has studied the steady flow of a viscous and incompressible fluid outside of a stretching hollow cylinder in an ambient fluid at rest. The problem is governed by a third-order nonlinear ordinary differential equation that leads to an exact similarity solutions of the Navier–Stokes equations. Motivated by the works of the above-mentioned authors [3–16], the present study considers the flow and heat transfer of a viscous and incompressible electrically conducting fluid outside of a stretching cylinder in the presence of a constant transverse magnetic field. The applications include fibre coating, metal spinning, wire drawing, flow meter design, piping and casting systems, etc. The problem is formulated in such a manner that the partial differential equations governing the flow and temperature fields are reduced to ordinary differential equations, which are solved analytically using homotopy analysis method (HAM).

These scientific problems are modeled by ordinary or partial differential equations. These equations should be solved using special techniques, because in most cases, analytical solutions cannot be applied to these problems. In recent years, much attention has been devoted to newly developed methods to construct an analytical solution of these equations; such methods include the Adomian decomposition method [17], Homotopy Perturbation Method [18, 19], Variational Iteration Method [20] and Perturbation techniques. Perturbation techniques are too strongly dependent upon the so-called 'small parameters' [21]. Thus, it is worthwhile to develop some new analytic techniques independent of small parameters. One of these techniques is HAM, which was introduced by Liao [22–28]. This method has been successfully applied to solve many types of nonlinear problems [29–38].

The problem under discussion is depicted in Figure 1.

2. THE BASIC IDEA OF HOMOTOPY ANALYSIS METHOD

Let us assume the following nonlinear differential equation in the form of:

$$N[u(\tau)]=0 \tag{1}$$

where N is a nonlinear operator, τ is an independent variable and $u(\tau)$ is the solution of equation. We define the function, $\phi(\tau, p)$, as follows:

$$\lim_{p\to 0}\phi(\tau, p)=u_0(\tau) \tag{2}$$

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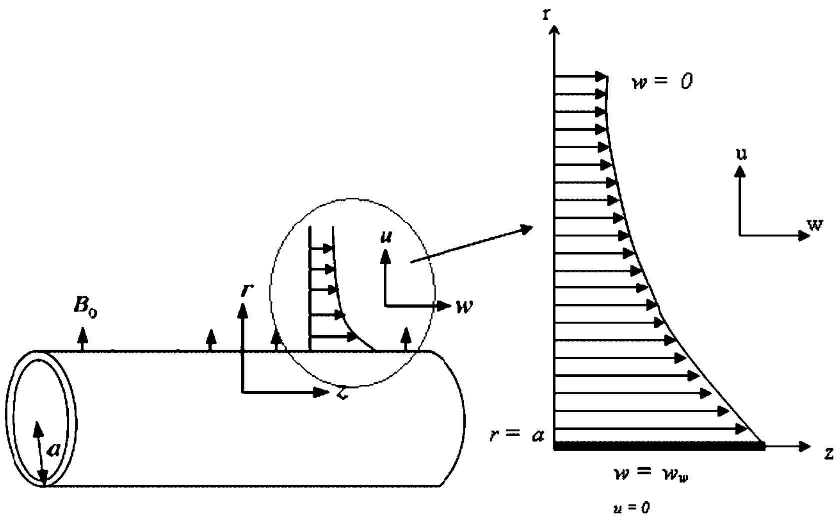

Figure 1. Schematic figure of the problem under discussion.

where $p \in [0, 1]$ and $u_0(\tau)$ is the initial guess which satisfies the initial or boundary condition and if

$$\lim_{p \to 1} \phi(\tau, p) = u(\tau) \tag{3}$$

and using the generalized homotopy method, Liao's so-called zero-order deformation equation will be:

$$(1-p)L[\phi(\tau, p) - u_0(\tau)] = p\hbar H(\tau)N[\phi(\tau, p)] \tag{4}$$

where $\hbar$ is the auxiliary parameter which helps us increase the results' convergence, $H(\tau)$ is the auxiliary function and L is the linear operator. It should be noted that there is a great freedom to choose the auxiliary parameter $\hbar$, the auxiliary function $H(\tau)$, the initial guess $u_0(\tau)$ and the auxiliary linear operator L. This freedom plays an important role in establishing the keystone of validity and flexibility of HAM as shown in this paper.

Thus, when p increases from 0 to 1 the solution $\phi(\tau, p)$ changes between the initial guess $u_0(\tau)$ and the solution $u(\tau)$. The Taylor series expansion of $\phi(\tau, p)$ with respect to p is:

$$\phi(\tau, p) = u_0(\tau) + \sum_{m=1}^{+\infty} u_m(\tau) p^m \tag{5}$$

and

$$u_0^{[m]}(\tau) = \left.\frac{\partial^m \phi(\tau; p)}{\partial p^m}\right|_{p=0} \tag{6}$$

where $u_0^{[m]}(\tau)$ for brevity is called the mth order of deformation derivation which reads:

$$u_m(\tau) = \frac{u_o^{[m]}}{m!} = \frac{1}{m!} \left.\frac{\partial^m \phi(\tau; p)}{\partial p^m}\right|_{p=0} \tag{7}$$

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It is clear that if the auxiliary parameter is $\hbar = -1$ and the auxiliary function is determined to be $H(\tau) = 1$, Equation (1) will be:

$$(1-p)L[\phi(\tau, p) - u_0(\tau)] + p(\tau)N[\phi(\tau, p)] = 0 \tag{8}$$

This statement is commonly used in the HPM procedure. In deed, in HPM we solve the nonlinear differential equation by separating any Taylor expansion term. Now we define the vector of:

$$\mathbf{u}_m = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_n\} \tag{9}$$

According to the definition in Equation (7), the governing equation and the corresponding initial condition of $u_m(\tau)$ can be deduced from zero-order deformation equation (1). Differentiating Equation (1) for m times with respect to the embedding parameter p and setting $p = 0$ and finally dividing by $m!$, we will have the so-called mth-order deformation equation in the form:

$$L[u_m(\tau) - \chi_m u_{m-1}(\tau)] = \hbar H(\tau) R(\mathbf{u}_{m-1}) \tag{10}$$

where

$$R_m(\mathbf{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(\tau; p)]}{\partial p^{m-1}}\Bigg|_{p=0} \tag{10a}$$

and

$$\chi_m = \begin{cases} 0, & m \leqslant 1 \\ 1, & m > 1 \end{cases} \tag{10b}$$

Hence, by applying inverse linear operator to both sides of the linear equation, Equation (1), we can easily solve the equation and compute the generation constant by applying the initial or boundary condition.

3. DESCRIPTION OF THE PROBLEM

Steady laminar flow of an incompressible electrically conducting fluid (with electrical conductivity σ) caused by a stretching tube of radius a in the axial direction in a fluid at rest is shown in Figure 1, where z is the axis along the tube length and r is the axis in the radial direction. The surface of the tube is at constant temperature T_w and the ambient fluid temperature is T_1, where $T_w > T_1$. Uniform magnetic field of intensity B_0 acts in the radial direction and the effect of the induced magnetic field is negligible, which is valid when the magnetic Reynolds number is small. The viscous dissipation, Ohmic heating, and Hall effects are neglected as they are also assumed to be small. Under these assumptions, the governing equations are [14, 15]

$$\frac{\partial}{\partial z}(rw) + \frac{\partial}{\partial r}(ru) = 0 \tag{11}$$

$$w\frac{\partial w}{\partial z} + u\frac{\partial w}{\partial r} = v\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \frac{\sigma B_0^2}{\rho} w \tag{12}$$

DOI: 10.1002/fld

$$w\frac{\partial u}{\partial z}+u\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\upsilon\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right) \tag{13}$$

$$w\frac{\partial T}{\partial z}+u\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{14}$$

subject to the boundary condition

$$\begin{aligned} &u=0, \quad w=w_w, \quad T=T_w \quad \text{at } r=a \\ &w\to 0, \quad T\to T_\infty \quad \text{as } r\to\infty \end{aligned} \tag{15}$$

where u and w are the velocity components along the r and z directions, respectively, and $w_w=2cz$ where c is a constant with positive value. Further υ, ρ, T, and α are, respectively, the kinematic viscosity, fluid density, fluid temperature, and thermal diffusivity.

Following Wang [14] we take the similarity transformation

$$\begin{aligned} &\eta=\left(\frac{r}{a}\right)^2, \quad u=-ca\frac{f(\eta)}{\sqrt{\eta}} \\ &w=2cf'(\eta)z, \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \end{aligned} \tag{16}$$

where prime denotes differentiation with respect to η. Substituting Equation (16) into Equations (12) and (15), we get the following ordinary differential equations:

$$Re(f'^2-ff'')=\eta f'''+f''-Mf' \tag{17}$$

$$\eta\theta''+(1+Re\,Pr f)\theta'=0 \tag{18}$$

where $Re=ca^2/(2\nu)$ is the Reynolds number and $M=\sigma B_0^2a^2/4\nu\rho$ is the magnetic parameter. The boundary conditions in Equation (15) become

$$\begin{aligned} &f(1)=0, \quad f'(1)=1, \quad \theta(1)=1 \\ &f'(\infty)\to 0, \quad \theta(\infty)\to 0 \end{aligned} \tag{19}$$

The pressure can now be determined from Equation (13) in the form

$$\frac{P-P_\infty}{r\upsilon c}=-\frac{Re}{\eta}f^2(\eta)-2f'(\eta) \tag{20}$$

The physical quantities of interest are the skin friction coefficient and the Nusselt number, which are defined as

$$C_f=\frac{\tau_w}{\rho w_w^2/2}, \quad Nu=\frac{aq_w}{k(T_w-T_\infty)} \tag{21}$$

with k being the thermal conductivity. Further, τ_w and q_w are the skin friction and the heat transfer from the surface of the tube, respectively, and they are given by

$$\tau_w=\mu\left(\frac{\partial w}{\partial r}\right)_{r=a}, \quad q_w=-k\left(\frac{\partial T}{\partial r}\right)_{r=a} \tag{22}$$

DOI: 10.1002/fld

Using Equation (16), we get

$$C_f(Rez/a) = f''(1), \quad Nu = -2\theta'(1) \tag{23}$$

4. SOLUTION USING HOMOTOPY ANALYSIS METHOD

In this section, we employ HAM to solve Equations (17) and (18) subject to boundary conditions in Equation (19). For solutions, we choose the initial guesses and auxiliary linear operators in the following form:

$$f_0(\eta) = 1 - ee^{-\eta}, \quad \theta_0(\eta) = ee^{-\eta} \tag{24}$$

As the initial guess approximation for $f(\eta)$ and $\theta(\eta)$

$$L_1(F) = f''' - f'', \quad L_2(\theta) = \theta'' - \theta' \tag{25}$$

As the auxiliary linear operator which has the property:

$$L(c_1\eta + c_2 + c_3 e^{-\eta}) = 0, \quad L(c_4 + c_5 e^{-\eta}) = 0 \tag{26}$$

and c_1–c_5 are constants. Let $p \in [0, 1]$ denotes the embedding parameter and $\hbar$ indicates non-zero auxiliary parameters. Then, we construct the following equations:

4.1. Zeroth-order deformation equations

$$(1-p)L_1[f(\eta; p) - f_0(\eta)] = p\hbar_1 N_1[f(\eta; p)] \tag{27}$$

$$(1-p)L_2[\theta(\eta; p) - \theta_0(\eta)] = p\hbar_2 N_2[\theta(\eta; p)] \tag{28}$$

$$f(1; p) = 0, \quad f'(1; p) = 1, \quad f'(\infty; p) = 0 \tag{29}$$

$$\theta(1; p) = 1, \quad \theta(\infty; p) = 0 \tag{30}$$

$$\begin{aligned} N_1[f(\eta; p)] = {} & \eta \frac{d^3 f(\eta; p)}{d\eta^3} + \frac{d^2 f(\eta; p)}{d\eta^2} - M \frac{df(\eta; p)}{d\eta} \\ & - Re\left(\left(\frac{df(\eta; p)}{d\eta}\right)^2 - f(\eta; p)\frac{d^2 f(\eta; p)}{d\eta^2}\right) = 0 \end{aligned} \tag{31}$$

$$N_2[\theta(\eta; p)] = \eta \frac{d^2\theta(\eta; p)}{d\eta^2} + (1 + RePrf(\eta; p))\frac{d\theta(\eta; p)}{d\eta} = 0 \tag{32}$$

For $p = 0$ and $p = 1$:

$$f(\eta; 0) = f_0(\eta), \quad f(\eta; 1) = f(\eta), \quad \theta(\eta; 0) = \theta_0(\eta), \quad \theta(\eta; 1) = \theta(\eta) \tag{33}$$

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When p increases from 0 to 1 then $f(\eta; p)$ vary from $f_0(\eta)$ to $f(\eta)$ and $\theta(\eta; p)$ vary from $\theta_0(\eta)$ to $\theta(\eta)$. By Taylor's theorem and using Equation (33), we can write:

$$f(\eta; p) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta) p^m, \quad f_m(\eta) = \frac{1}{m!} \frac{\partial^m (f(\eta; p))}{\partial p^m} \tag{34}$$

$$\theta(\eta; p) = \theta_0(\eta) + \sum_{m-1}^{\infty} \theta_m(\eta) p^m, \quad \theta_m(\eta) = \frac{1}{m!} \frac{\partial^m (\theta(\eta; p))}{\partial p^m} \tag{35}$$

For simplicity, we suppose $\hbar_1 = \hbar_2 = \hbar$, in which $\hbar$ is chosen in such a way that these two series are convergent at $p = 1$. Therefore, we have through Equations (34) and (35):

$$f(\eta) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta) \tag{36}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m-1}^{\infty} \theta_m(\eta) \tag{37}$$

4.2. *mth-order deformation equations*

$$L[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar R_m^f(\eta) \tag{38}$$

$$f_m(1) = f'_m(1) = f'_m(\infty) = 0 \tag{39}$$

$$R_m^f(\eta) = \eta f'''_{m-1} - f''_{m-1} + M f'_{m-1} + \sum_{n=0}^{m-1} Re(f_{m-1-n} f''_n - f'_{m-1-n} f'_n) \tag{40}$$

$$L[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar R_m^\theta(\eta) \tag{41}$$

$$\theta_m(1) = \theta_m(\infty) = 0 \tag{42}$$

$$R_m^\theta(\eta) = \eta \theta''_{m-1} + \theta'_{m-1} + \sum_{n=0}^{m-1} Re Pr f_{m-1-n} \theta'_n \tag{43}$$

$$\chi_m = \begin{cases} 0, & m \leqslant 1 \\ 1, & m > 1 \end{cases} \tag{44}$$

To obtain the solutions of Equations (17) and (18) subject to boundary conditions (19) up to first few orders of approximations the series solution is found to be:

$$f_m(\eta) = a_0^{m,0} + \sum_{n=1}^{m+1} e^{-n\eta} \sum_{k=0}^{2(m+1-n)} \eta^k a_k^{m,n} \tag{45}$$

$$\theta_m(\eta) = b_0^{m,0} + \sum_{n=1}^{m+1} e^{-n\eta} \sum_{k=0}^{2(m+1-n)} \eta^k b_k^{m,n} \tag{46}$$

Substituting Equations (45)–(46) into Equations (38)–(43), the recurrence formulae for the coefficient $a_k^{m,n}$ and $b_k^{m,n}$ of $F_m(\eta)$ and $\theta_m(\eta)$ are obtained, respectively, for $m \geqslant 1, 0 \leqslant n \leqslant m+1$ and

$0 \leqslant k \leqslant 2(m+1-n)$ as:

$$a_0^{m,0} = \chi_m a_0^{m-1,0} - \sum_{r=0}^{2m-1} \left[\Gamma 1_r^{m,1} \Pi 1_r^{1,1} - \sum_{n=2}^{m+1} (n-1) \Gamma 1_0^{m,n} \Pi 1_0^{n,0} + \sum_{r=1}^{2(m-n+1)} \Gamma 1_r^{m,n} ((n-1) \Pi 1_r^{n,0} - \Pi 1_r^{n,1}) \right] \tag{47}$$

$$a_0^{m,1} = \chi_m a_0^{m-1,1} + \sum_{r=0}^{2m-1} \Gamma 1_r^{m,1} \Pi 1_r^{1,1} + \sum_{n=2}^{m+1} n \Gamma 1_0^{m,n} \Pi 1_0^{n,0} + \sum_{r=1}^{2(m-n+1)} \Gamma 1_r^{m,n} (n \Pi 1_r^{n,0} - \Pi 1_r^{n,1}) \tag{48}$$

$$a_k^{m,1} = \chi_m (1 - \chi_{k+3-2m}) a_k^{m-1,1} + \sum_{r=k-1}^{2m-1} \Gamma 1_r^{m,1} \Pi 1_r^{1,k} \quad (1 \leqslant k \leqslant 2m) \tag{49}$$

$$a_k^{m,n} = \chi_m (1 - \chi_{k+1-2m+2n}) a_k^{m-1,n} - \sum_{r=k}^{2(m-n+1)} \Gamma 1_r^{m,n} \Pi 1_r^{n,k} \tag{50}$$

where

$$\Pi 1_r^{1,k} = \frac{r!(r-k+2)}{k!} \tag{51}$$

$$\Pi 1_r^{n,k} = \frac{r!}{k!(n-1)^{r-k+1}} \left[1 - \left(1 - \frac{1}{n}\right)^{r-k+1} \left(1 + \frac{r-k+1}{n}\right) \right], \quad n \geqslant 2, 0 \leqslant k \leqslant r \tag{52}$$

$$\Gamma 1_r^{m,n} = \hbar_1 [\chi_{2(m-n)-r+2} \{ \eta a3_r^{m-1,n} - a2_r^{m-1,n} - M a1_r^{m-1,n} - Re(\delta 1_r^{m-1,n} - \delta 2_r^{m-1,n}) \}], \quad (1 \leqslant n \leqslant m, 0 \leqslant r \leqslant 2m-2n+2) \tag{53}$$

For $\theta(\eta)$ we have the recurrence formulae as:

$$b_k^{m,n} = \chi_m (1 - \chi_{k+1-2m+2n}) b_k^{m-1,n} - \sum_{r=k}^{2(m-n+1)} \Gamma 2_r^{m,n} \Pi 2_r^{n,k} \tag{54}$$

$$b_0^{m,1} = \chi_m (1 - \chi_{3-2m}) b_0^{m-1,1} - \sum_{r=0}^{2m} \Gamma 2_r^{m,1} \Pi 2_r^{1,0} \tag{55}$$

$$b_k^{m,1} = \chi_m (1 - \chi_{k+3-2m}) b_k^{m-1,1} - \sum_{r=k}^{2n} \Gamma 2_r^{m,1} \Pi 2_r^{1,k} \tag{56}$$

where

$$\Pi 2_r^{1,k} = \frac{r!(r-k+2)}{k!} \tag{57}$$

$$\Pi 2_r^{n,k} = \frac{r!}{k!(n-1)^{r-k+1}} \left[1 - \left(1 - \frac{1}{n}\right)^{r-k+1} \left(1 + \frac{r-k+1}{n}\right) \right], \quad n \geqslant 2, 0 \leqslant k \leqslant r \tag{58}$$

DOI: 10.1002/fld

$$\Gamma 2_r^{m,n} = \hbar_2[\chi_{2(m-n)-r+2}\{\eta b2_r^{m-1,n} + 2b2_r^{m-1,n} + RePr\delta 3_r^{m-1,n}\}], \quad (1\leqslant n\leqslant m, 0\leqslant r\leqslant 2m-2n+2) \tag{59}$$

And the coefficient $\delta i_r^{m,n}$ and $i=1$ to 7 when $m\geqslant 1$, $0\leqslant n\leqslant m+1$, $0\leqslant r\leqslant 2(m+1-n)$ are:

$$\delta 1_r^{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max(0,n-m+k)}^{\min(n,k+1)} \sum_{i=\min(0,q-2(m-k-n+j))}^{\min(q,2(k+1-j))} a1_{r-i}^{m-1-k,n-j} a1_i^{k,j} \tag{60}$$

$$\delta 2_r^{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max(0,n-m+k)}^{\min(n,k+1)} \sum_{i=\min(0,q-2(m-k-n+j))}^{\min(q,2(k+1-j))} a_{r-i}^{m-1-k,n-j} a3_i^{k,j} \tag{61}$$

$$\delta 3_r^{m,n} = \sum_{k=0}^{m-1} \sum_{j=\max(0,n-m+k)}^{\min(n,k+1)} \sum_{i=\min(0,q-2(m-k-n+j))}^{\min(q,2(k+1-j))} a_{r-i}^{m-1-k,n-j} b1_i^{k,j} \tag{62}$$

where:

$$a1_k^{m,n} = (k+1)a_{k+1}^{m,n} - n\, a_k^{m,n} \tag{63}$$

$$a2_k^{m,n} = (k+1)a1_k^{m,n} - n\, a1_k^{m,n} \tag{64}$$

$$a3_k^{m,n} = (k+1)a2_{k+1}^{m,n} - n\, a2_k^{m,n} \tag{65}$$

$$b1_k^{m,n} = (k+1)b_{k+1}^{m,n} - n\, b_k^{m,n} \tag{66}$$

Given by the initial guess approximation in Equation (47), the corresponding mth-order approximation of Equations (42)–(43) and (45)–(46) is then given by:

$$F(\eta) = \lim\left(\sum_{m=0}^{M} a_0^{m,0} + \sum_{n=1}^{M+1} e^{-n\eta}\left(\sum_{m=n-1}^{M}\sum_{k=0}^{2(m+1-n)} \eta^k a_k^{m,n}\right)\right) \tag{67}$$

$$\theta(\eta) = \lim\left(\sum_{m=0}^{M} b_0^{m,0} + \sum_{n=1}^{M+1} e^{-n\eta}\left(\sum_{m=n-1}^{M}\sum_{k=0}^{2(m+1-n)} \eta^k b_k^{m,n}\right)\right) \tag{68}$$

5. CONVERGENCE OF THE HAM SOLUTION

As was mentioned in the introduction, HAM provides us with great freedom in choosing the solution of a nonlinear problem by different base functions. This has a great effect on the convergence region because the convergence region and the rate of a series are chiefly determined by the base functions used to express the solution. Therefore, we can approximate a nonlinear problem more efficiently by choosing a proper set of base functions and ensure its convergency. On the other hand, as pointed out by Liao, the convergence and rate of approximation for the HAM solution strongly depend on the value of the auxiliary parameter. By means of the so-called $\hbar$-curves, it is easy to find out the so-called valid regions of the auxiliary parameters to gain a convergent solution series.

The $\hbar$ region for this problem is shown in Figures 2 and 3.

DOI: 10.1002/fld

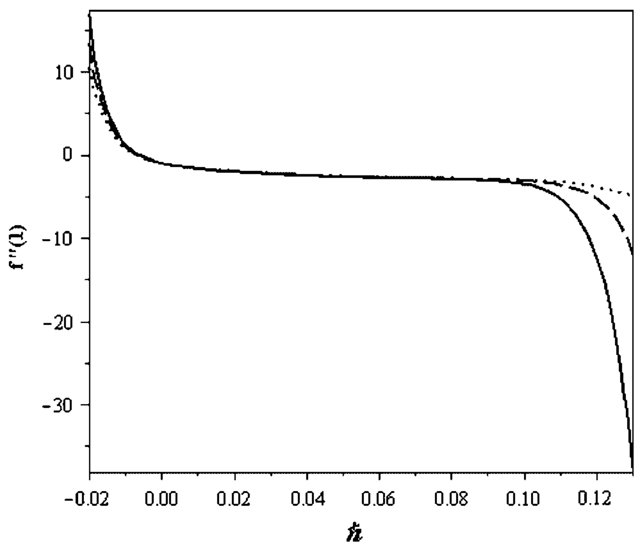

Figure 2. The $\hbar$-validity curve for $f''(1)$, solid curve: 22th-order approximate, dashed curve: 21th-order approximate and dotted curve: 20th-order approximate when $M=0$ and $Re=10$.

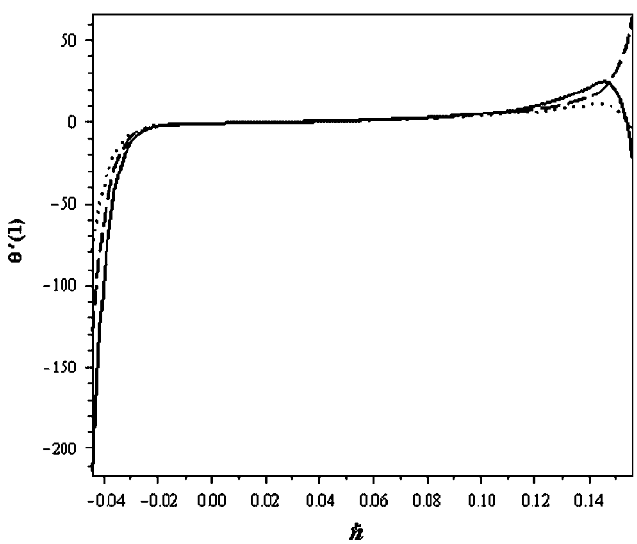

Figure 3. The $\hbar$-validity curve for $\theta'(1)$, solid curve: 22th-order approximate, dashed curve: 21th-order approximate and dotted curve: 20th-order approximate when $M=0$, $Re=10$ and $Pr=0.7$.

6. RESULTS AND DISCUSSION

The results are shown in Figures 4–8. These figures show influences of several non-dimensional parameters, namely the Reynolds number Re, the Prandtl number Pr, and the magnetic parameter M.

DOI: 10.1002/fld

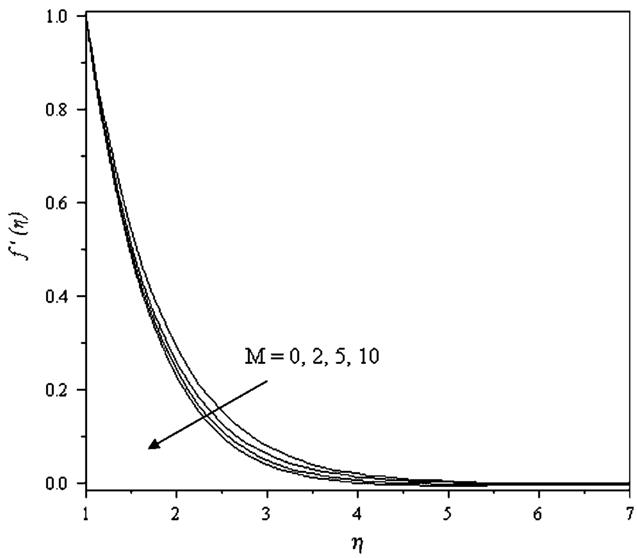

Figure 4. Velocity profile $f'(\eta)$ for various values of M when $Re = 10$.

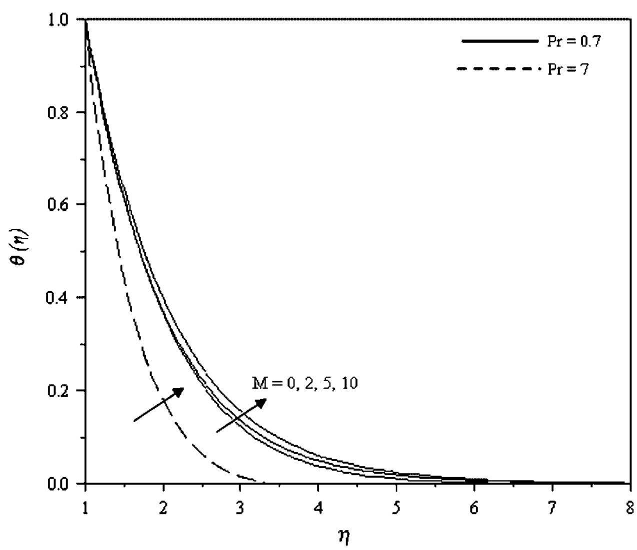

Figure 5. Temperature profile $\theta(\eta)$ for various values of M and Pr when $Re = 10$.

Figure 4 shows the velocity profiles for various values of the magnetic parameter M when $Re = 10$. It is noticed that the Prandtl number Pr gives no effect to the velocity as can be seen from Equation (17). The velocity curves show that the rate of transport is considerably affected and reduced with the increase of M.

DOI: 10.1002/fld

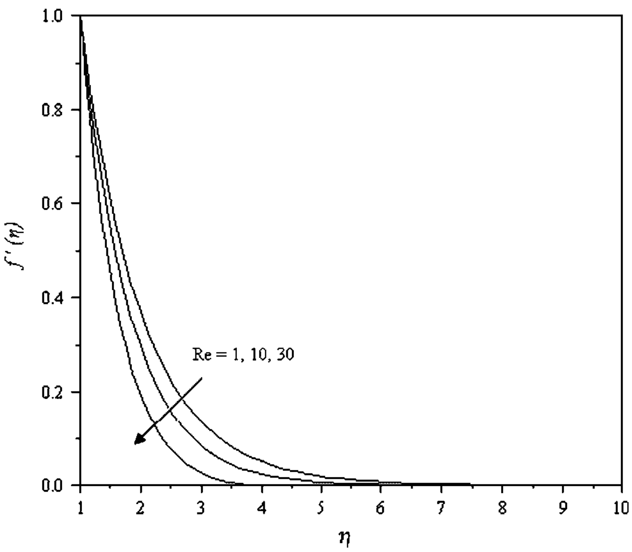

Figure 6. velocity profile $f'(\eta)$ for various values of Re when $M=0.1$.

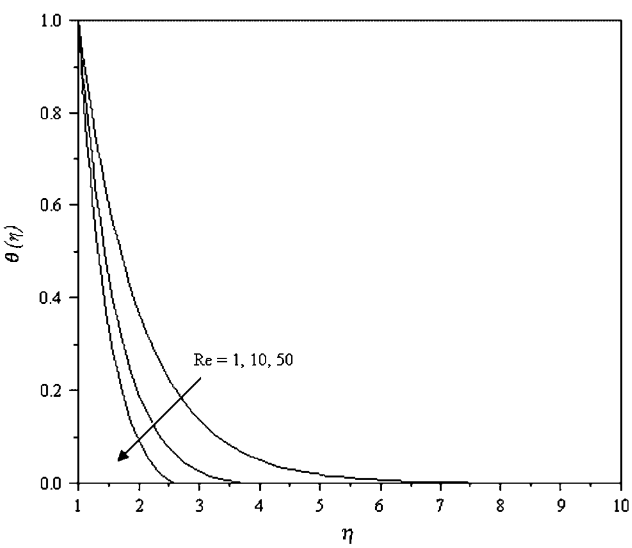

Figure 7. Temperature profile $\theta(\eta)$ for various values of Re when $M=0.1$ and $Pr=7$.

Figure 5 presents the temperature profiles for various values of M and Pr when $Re=10$. For both $Pr=0.7$ (such as air) and $Pr=7$ (such as water), the temperature is found to increase as M increases, but it decreases as the distance from the surface increases, and finally vanishes at some large distance from the surface. The effect of M is found to be more pronounced for fluids

DOI: 10.1002/fld

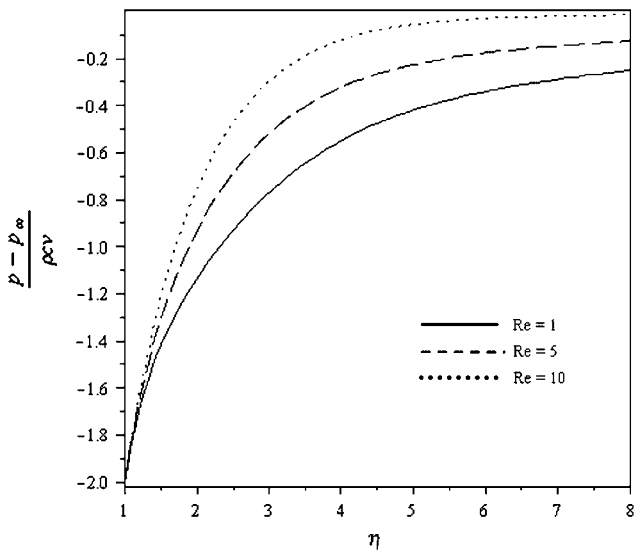

Figure 8. pressure distribution $(p-p_\infty)/\rho vc$ when $Re=1$.

with smaller Pr since they have a larger thermal diffusivity. Thus, fluids having a smaller Pr are more sensitive to the magnetic force than those with a larger Pr.

Figures 6 and 7 exhibit the velocity and temperature profiles, respectively, for various values of the Reynolds number Re. It is observed that both velocity and temperature profiles decrease as Re increases, which shows similar results as those of Wang [18] for non-magnetic case. It is obvious that the Reynolds number indicates the relative significance of the inertia effect compared with the viscous effect. The velocity and temperature vanish at some large distance from the surface of the tube. From Figure 6, it is clear that the velocity boundary layer thickness decreases as Re increases which implies an increase in the velocity gradient, and hence increase in the magnitude of the skin friction coefficient. After velocity $f'(\eta)$ is obtained, the pressure P in terms of $(P-P_\infty)/\rho vc$ can be found by using Equation (11). The HAM results are shown in Figure 6 for $Re=0.1$, 1, and 10. All the curves show that $p \to p_\infty$ is far away from the surface $\eta \to \infty$.

7. CONCLUSIONS

The similarity solutions to the governing equations of the steady two-dimensional flow of an electrically conducting incompressible fluid due to a stretching cylindrical tube have been obtained using HAM. The effects of the magnetic parameter, the Prandtl number, and the Reynolds number on the flow and heat transfer characteristics have been studied. From this investigation it is concluded that transverse magnetic field decreases the velocity field, but this is inverse in results for temperature.

These results are obtained by HAM. This method provides highly accurate numerical solutions for nonlinear problems in comparison with other methods. The auxiliary parameter $\hbar$ provides us with a convenient way to adjust and control the convergence and its rate for the solutions series.

DOI: 10.1002/fld

Finally, it has been attempted to show the capabilities and wide-range applications of the HAM of steady two-dimensional flow of an electrically conducting incompressible fluid due to a stretching cylindrical tube.

NOMENCLATURE

HAM	homotopy analysis method
a	radius of cylinder
B_0	uniform magnetic field
c	positive constant
C_f	skin friction coefficient
f	dimensionless stream function
k	thermal conductivity
M	magnetic parameter
Nu	Nusselt number
P	pressure
Pr	Prandtl number
q_w	heat transfer from the cylinder surface
Re	Reynolds number
T	fluid temperature
T_w	temperature of the cylinder surface
$T_{¥}$	ambient temperature
u, w	velocity component in the r, z directions
r, z	cylindrical coordinate in the radial and axial direction
w_w	velocity of the stretching cylinder
α	thermal diffusivity
η	similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
υ	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
ψ	stream function
τ_w	skin friction

REFERENCES

1. Liron N, Wilhelm HE. Integration of the magneto-hydrodynamic boundary layer equations by Meksyn's method. *Journal of Applied Mathematics and Mechanics* (*ZAMM*) 1974; **54**:27–37.
2. Pavlov KB. Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. *Magnitnaya Gidrodinamika* 1974; **4**:146–147.
3. Chakrabarti A, Gupta AS. Hydromagnetic flow and heat transfer over stretching sheet. *Quarterly of Applied Mathematics* 1979; **37**:73–78.
4. Takhar HS, Raptis AA, Perdikis CP. MHD asymmetric flow past a semi-infinite moving plate. *Acta Mechanica* 1987; **65**:287–290.

DOI: 10.1002/fld

5. Chiam TC. Magnetohydrodynamic boundary layer flow due to a continuously moving plate. *Computers and Mathematics with Applications* 1993; **26**:1–7.
6. Watanabe T, Pop I, Gota F. MHD stability of boundary layer flow over a moving flat plate. *Technische Mechanik* 1995; **15**:325–332.
7. Chiam TC. Hydromagnetic flow over a surface stretching with a power-law velocity. *International Journal of Engineering Science* 1995; **33**:429–435.
8. Lawrence PS, Rao BN. Magnetohydrodynamic flow past a semi-infinite moving plate. *Acta Mechanica* 1996; **117**:159–164.
9. Chiam TC. Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet. *Acta Mechanica* 1997; **122**:169–179.
10. Vajravelu K, Hadjinicolaou A. Convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. *International Journal of Engineering Science* 1997; **35**:1237–1244.
11. Mahapatra TR, Gupta AS. Magnetohydrodynamic stagnation-point flow towards a stretching sheet. *Acta Mechanica* 2001; **152**:191–196.
12. Ishak A, Nazar R, Pop I. Magnetohydrodynamic stagnation-point flow towards a stretching vertical sheet. *Magnetohydrodynamics* 2006; **42**:17–30.
13. Magyari E, Keller B. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *Journal of Physics D, Applied Physics* 1999; **32**:577–585.
14. Pantokratoras A. Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity: A numerical reinvestigation. *International Journal of Heat and Mass Transfer* 2008; **51**(1–2):104–110.
15. Mamun AA, Chowdhury ZR, Azim MA, Molla MM. MHD-conjugate heat transfer analysis for a vertical flat plate in presence of viscous dissipation and heat generation. *International Communications in Heat and Mass Transfer* 2008; **35**(10):1275–1280.
16. Wang CY. Fluid flow due to a stretching cylinder. *Physics of Fluids* 1988; **31**:466–468.
17. Adomian G. A review of the decomposition method and some recent results for nonlinear equation. *Mathematical and Computer Modelling* 1992; **13**(7):17.
18. Ganji DD, Rafei M, Sadighi A, Ganji ZZ. A comparative comparison of He's method with perturbation and numerical methods for nonlinear vibrations equations. *International Journal of Nonlinear Dynamics in Engineering and Sciences* 2009; **1**(1):1–20.
19. Rashidi MM, Diranvand S. Explicit and analytical traveling wave solutions of Whitham–Broer–Kaup shallow water equations by homotopy perturbation method. *International Journal of Nonlinear Dynamics in Engineering and Sciences* 2009; **1**(1):99–107.
20. Abbasbandy S, Shirzadi A. The variational iteration method for a family of fifth-order boundary value differential equations. *International Journal of Nonlinear Dynamics in Engineering and Sciences* 2009; **1**(1):39–46.
21. Nayfeh AH. *Perturbation Methods*. Wiley: New York, 2000.
22. Liao SJ. The proposed homotopy analysis technique for the solution of nonlinear problems. *Ph.D. Thesis*, Shanghai Jiao Tong University, 1992.
23. Liao SJ. An approximate solution technique not depending on small parameters: a special example. *International Journal of Non-Linear Mechanics* 1995; **303**:371–380.
24. Liao SJ. Boundary element method for general nonlinear differential operators. *Engineering Analysis with Boundary Elements* 1997; **202**:91–99.
25. Liao SJ. An explicit, totally analytic approximate solution for Blasius' viscous flow problems. *International Journal of Non-Linear Mechanics* 1999; **34**:759–778.
26. Liao SJ. On the analytic solution of magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet. *Journal of Fluid Mechanics* 2003; **488**:189–212.
27. Liao SJ. On the homotopy analysis method for nonlinear problems. *Applied Mathematics and Computation* 2004; **147**:499–513.
28. Liao SJ. A new branch of solutions of boundary-layer flows over an impermeable stretched plate. *International Journal of Heat and Mass Transfer* 2005; **48**(12):2529–2539.
29. Hayat T, Khan M, Ayub M. Couett and Poisevill flow of an oldroyd 6-constant fluid with magnetic field. *Journal of Mathematical Analysis and Applications* 2004; **298**:225–244.
30. Hayat T, Khan M, Asghar S. Homotopy analysis MHD flows of an oldroyd 8-constant fluid. *Acta Mechanica* 2004; **168**:213–232.
31. Hayat T, Khan M. Homotopy solutions for generalized second-grade fluid past porous plate. *Nonlinear Dynamics* 2005; **42**:395–405.

DOI: 10.1002/fld

32. Fakhari A, Domairry G. Ebrahimpour Approximate explicit solutions of nonlinear BBMB equations by homotopy analysis method and comparison with the exact solution. *Physics Letters A* 2007; **368**:64–68.
33. Domairry G, Nadim N. Assessment of homotopy analysis method and homotopy perturbation method in non-linear heat transfer equation. *International Communications in Heat and Mass Transfer* 2008; **35**(1):93–102.
34. Rashidi MM, Domairry G, Dinarvand S. Solution of the laminar viscous flow in a semi-porous channel in the presence of a uniform magnetic field by using the homotopy analysis method. *Communications in Nonlinear Science and Numerical Simulation* 2007; **14**:708–717.
35. Abbasbandy S. The application of homotopy analysis method to solve a generalized Hirota–Satsuma coupled KdV equation. *Physics Letters A* 2007; **361**(6):478–483.
36. Abbasbandy S. The application of homotopy analysis method to nonlinear equations arising in heat transfer. *Physics Letters A* 2006; **360**(1):109–113.
37. Abbasbandy S. Solitary smooth hump solutions of the Camassa–Holm equation by means of the homotopy analysis method. *Chaos*, *Solitons and Fractals* 2008; **36**(3):581–591.
38. Ganesan P, Loganathan P. Magnetic field effect on a moving vertical cylinder with constant heat flux. *Heat and Mass Transfer* 2003; **39**:381–386.